The pool hits: an analysis to the route of balls in pool

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Abstract

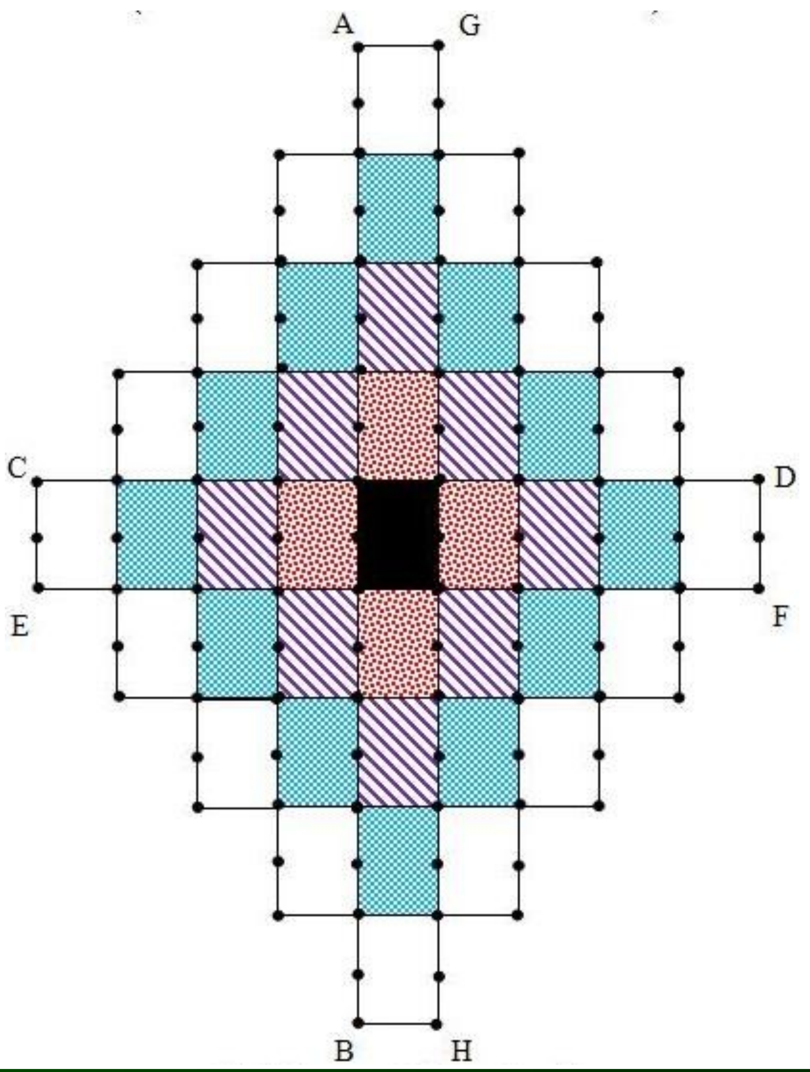
Our project studies the mathematics involved in the pool game. By considering physics and geometry, the routes of pool balls are calculated and the strategies to get target balls into pockets are proposed:

- Number of routes
- Direction of the route of object ball
- How contact point affects route of the cue ball
- Change of routes after collision

1 No. of routes

According to the reflection of force, after the ball hits the edges for n times, a similar diagram can be obtained in the right.

- The no. of routes for each of the four corner pockets is $(n+1)$
- The no. of routes for each of the two middle pocket is $(2n+1)$



2 Direction

If the ball rebounds at the horizontal cushions for i times and at the vertical cushions for j times (i and j are non-negative integers), we find that:

For the corner pockets:

When i and j are both even,

$$\theta = tan^{-1}\frac{b(i+1)-q}{aj+p}\ (0° < \theta < 90°)$$

When i is even and j is odd,

$$\theta = tan^{-1}\frac{-b(i+1)+q}{a(j+1)-p}\ (90° < \theta < 180°)$$

When i and j are both odd,

$$\theta = tan^{-1}\frac{bi+q}{a(j+1)-p}\ (180° < \theta < 270°)$$

When i is odd and j is even,

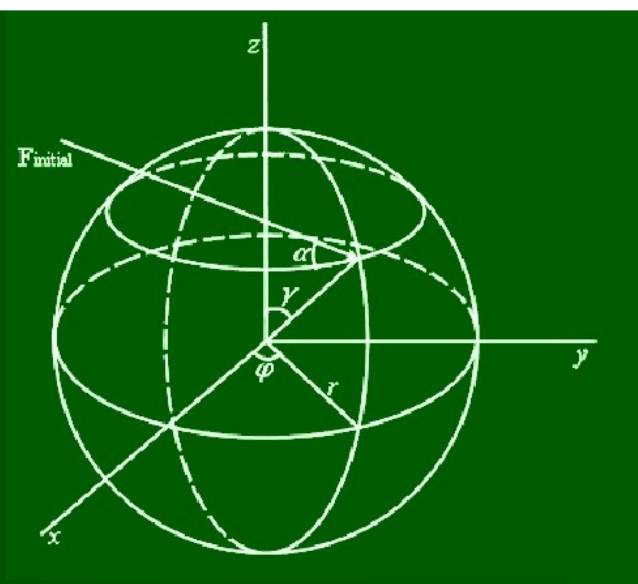
$$\theta = tan^{-1}\frac{-b(i+1)-q}{a(j-1)+p}\ (270° < \theta < 360°)$$

For the middle pockets:

Just replace every q in the above equations with $(q+1/2)$.

3 Cue Ball

Let the cue hit the ball at the point (r, φ, γ), which is in the right half of the ball with an angle of α to the horizon.



a) When the contact point is on the lower half of the ball, the displacement of the ball along the direction of y-axis according to time is

$$d_y = v_{initial}t + [K(W + cos\,\gamma\, F) - cos\,\beta\, \mu_0'(W + cos\,\gamma\, F)]t^2/2m$$

Similarly, the displacement of the ball along the direction of x-axis according to time is $d_x = sin\,\beta\, \mu_0'(W + cos\,\gamma\, F)t^2/2m$

b) When the contact point is on the upper half of the ball, the displacement of the ball along the direction of y-axis according to time is

$$d_y = v_{initial}t + [cos\,\beta\, \mu_0'(W + cos\,\gamma\, F) - K(W + cos\,\gamma\, F)]t^2/2m$$

Similarly, the displacement of the ball along the direction of x-axis according to time is $d_x = sin\,\beta\, \mu_0'(W + cos\,\gamma\, F)t^2/2m$

4 Collision

After the collision of two balls, the relative linear velocity at the point of contact of ball 1 with respect to ball 2 is found to be

$$\overrightarrow{v_{LR}} = r \times (\overrightarrow{\omega_1} - \overrightarrow{\omega_2})$$

The frictional force between the balls at the moment of collision is

$$\overrightarrow{F_f} = -\frac{\mu m \Delta v_{n1}}{\Delta t}\cdot\frac{\overrightarrow{v_{LR}} + \overrightarrow{v_{t1}}}{|\overrightarrow{v_{LR}} + \overrightarrow{v_{t1}}|}, \Delta v_{n1} = |\overrightarrow{v_{n1}}| + |\overrightarrow{v_{n2}}|$$

Since, $\overrightarrow{\tau_1} = I\overrightarrow{\alpha_1}$, $\overrightarrow{r_1} \times \overrightarrow{F_f} = \frac{2}{5}mr^2 \cdot \frac{\Delta\omega_1}{\Delta t}$

$$\Delta\omega_1 = \frac{5}{2}\overrightarrow{F_f}\frac{\Delta t}{mr^2}\overrightarrow{r_1}$$

Similarly, for ball 2, $\Delta\omega_2 = \frac{5}{2}\overrightarrow{F_f}\frac{\Delta t}{mr^2}\overrightarrow{r_2}$

Substitute the values into the result of part III

Displacement along the direction of the normal of contact is

$$d_y = |\overrightarrow{v_1}|t + (\mu_0' cos\,\beta - k)\frac{t^2\omega}{2m}$$

along the direction of the normal of contact is $d_x = \frac{t^2}{2m}\mu_0'\omega sin\,\beta$

Conclusion

- Routes of balls before and after collisions can be calculated accurately .
- A suitable strategy can be provided for players to win the game.

Future plan

- Further investigate the collision of three or more balls at one time.
- Explore the physics and mathematics of jump ball.

References

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