It's a small world after all.

An algorithmic approach to international flight network

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ABSTRACT

The phrase "small world effect" originates in an experiment done by Stanley Milgram in which he demonstrated the "six degrees of separation" – Any person on this globe is connected to any other person through no more than five intermediaries.

This project makes use of graph theory and computer programming to investigate the properties of the global airline network that involves 680 cities. Research is done through internet to determine whether a direct flight exists between any two cities. An adjacency matrix indicating such relationship is obtained. After computing the distance matrix from the adjacency matrix using Breadth First Search (BFS), the degree of separation of the network is found to be 4, proving the network to be a small world phenomenon. Codes are also written to find clustering coefficients, centralities and change in average distance after the removal of vertices.

The primary finding is that the network is a small world phenomenon due to its small mean vertex-to-vertex distance and high clustering coefficient. It is also a scale free network that contains nodes with a high degree, with the degree distribution follows a power law.. By investigating three centralities—degree centrality, closeness centrality and betweenness centrality, the more important nodes in this network are identified. The project also seeks to address other questions regarding the structure of the network such as the additional responsibilities that are to be shouldered by the other airports upon the removal of one vertex, in order to investigate the robustness of this network.

1. INTRODUCTION

Our inspiration for the project draws from the renowned phenomenon "small world effect". A classic example of the "small world effect" would be "six degrees of separation" in which everyone is estimated to be about six steps away from any other person on the earth,[1] demonstrating a rather robust social network. Subsequently, similar experiments were conducted to further illustrate the phenomenon: In 1978, American psychologist Stanley Milgram published his work on the United States network in which people are connected by approximately three friendship links.[2] In 2001, Duncan Watts recreated Milgram's experiment with a high tech approach: He used Email message as the "package" that needs to be delivered, with 48,000 senders and 19 targets in 157 countries. It turns out that the number of intermediaries is around 6.[3] This is quite an intriguing mystery to us and the project thus uses the international flight network as the target to study this small world effect.

The choice of the network stems primarily from the important role played by international flight network: For commercial giants, knowing the alpha city accessible from many other cities is crucial for strategising location of company to maximise the profits. For international agencies, a sense of robustness of the flight network helps to estimate the number of airports that can be closed when emergencies such as terrorism, influenza take place without a severe impact on the profits of airline companies. This is exemplified in the September 11 attack which has brought great loss to nearly all the airlines in the US, in particular, US Airways lost a combined 7.7 billion USD in 2001 and about 6 billion more in 2002.[4] The number of air passengers in China has decreased by nearly 50% within a year before due to the sudden eruption of severe acute respiratory syndrome (SARS).[5] The study will also facilitate the determination of socioeconomic positions of different cities from the perspective of accessibility.

While internal airline network within a country has been studied by various research groups with examples of papers including *Analysis and Optimization of airline networks: A Case Study of China*[6] and *Structure and External Factors of Chinese City Airline Network*[7], a comprehensive study on global flight network is still lacking. Hence, the project seeks to investigate the structural properties of this network from an algorithmic approach.

We hypothesise that the international flight network is a small world network that is robust.

1. RESEARCH METHODOLOGY

1.1. Data collection

In order to investigate the structural properties of the international flight network, we first searched on the Internet to find a list of cities across the globe that have at least one airport. A total of 680 cities (as shown in Appendix A) were chosen and these cities are the vertices in the graph representing the flight network. After which, we determined whether there exists a direct flight between each pair of cities in the entire year from a flight search website, www.skyscanner.net. If there is, the two vertices are then joined by an edge. All these data are represented by a 680×680 adjacency matrix, A.

1.2. Data processing – Computer programming

We did programming on MATLAB and some calculations to determine the followings from the adjacency matrix we have obtained earlier:

a) Distance matrix by performing breadth-first-search for each vertex
   Distance is the shortest path length, which in this case would be the minimum number of flights one has to take in order to travel from one city to another.
   Determine the diameter of the network, which is the greatest distance between any two vertices, and calculate the average distance between 2 vertices, $\bar{d}$.

b) Clustering coefficient[8]

i) Global clustering coefficient,

$$C = \frac{3 \times (number\ of\ triangles\ on\ the\ graph)}{(number\ of\ connected\ triples\ of\ vertices)}$$

$$= \frac{6 \times (number\ of\ triangles\ on\ the\ graph)}{(number\ of\ paths\ of\ length\ 2)}$$

where a triangle means three vertices that are each connected to both of the others, and a connected triple means a vertex that is connected to a pair of other vertices, which may or may not be connected to each other.

ii) Network average clustering coefficient,

$$\bar{C} = \frac{1}{n}\sum_{i=1}^{n} C_i = \frac{1}{n}\sum_{i=1}^{n} \frac{(number\ of\ connected\ neighbour\ pairs)}{\frac{1}{2}k_i(k_i - 1)}$$

where k_i is the degree of vertex i, n is the number of vertices in the network.

iii) Clustering coefficient of a random graph,

C_{rg} = probability that two vertices are connected = z / (n-1)

where z is the mean degree of a vertex.

The clustering coefficients of the network are compared with the coefficient of a random graph of the same size and mean degree.

c) Matrix W, where the (i,j) entry of W is the number of different $v_i - v_j$ walks of minimum length.

d) Centrality of each vertex

The centrality of a vertex is a measure of its structural importance in the network. There are three main types of centrality: degree, closeness, and betweenness. [9]

i) Degree centrality:

$$DC(v) = \frac{\deg(v)}{n-1}$$

where $\deg(v)$, the degree of a vertex, is the number of edges incident to it.

ii) Closeness centrality:

$$CC(v) = \frac{n-1}{\sum_{v \neq u} d(v,u)}$$

iii) Betweenness centrality: [10]

$$BC(v) = \sum_{\substack{u,w \in V \\ u \neq v \neq w}} \frac{\sigma_{uw(v)}}{\sigma_{uw}}$$

where σ_{uw} is the number of shortest paths from *u* to *w*, and $\sigma_{uw(v)}$ is the number of shortest paths from *u* to *w* that pass through the vertex v.

e) Remove one vertex or a set of vertices (vertices with the highest centrality that are identified above and vertices chosen at random). Obtain the distance matrix of the resulting network and determine the diameter as well as the average distance.

The MATLAB codes that we have written to perform the above functions are in Appendix B.

1.3. Data analysis

After obtaining the data as shown above, several analyses would be conducted to study the structural properties of the international flight network:

a) Investigate the small world properties of the network by analysing the statistics regarding the network (eg diameter, mean distance and clustering coefficients) and comparing these values with that of a random graph with the same number of vertices and mean degree.
b) Analyse the distribution of the degrees.
c) Compare and analyse the different centralities of the vertices. Determine the cities with high centrality which would be the so-called 'hubs' or the important nodes in the network.
d) Examine the robustness of the network by determining the diameter, mean distance, number of disconnected components of the resulting networks and compare these data with that of the original network.

2. RESULTS AND DISCUSSION

2.1. Small world network

Table 1 shows the statistics of the international flight network. From the data, we can make some deductions on the structural properties of the network.

International flight network	Diameter	Mean vertex-to-vertex distance $\bar{d}$	Global clustering coefficient C	Network average clustering coefficient $\bar{C}$	Clustering coefficient of a random graph C_{random}
	8	2.572	0.444	0.592	0.041

Table 1: Diameter, mean distance $\bar{d}$, global clustering coefficient C, network average clustering coefficient $\bar{C}$ of the international flight network, along with the expected value of the clustering coefficient of a random graph C_{random} with the same number of vertices (680) and the same mean degree (27.909).

In this network, while most vertices are not adjacent to one another, most vertices can be reached from another in 2 or 3 steps and the largest number of steps from remote vertices (cities) is at most 8. Furthermore, the clustering coefficient of the network is found to be much greater than that of a random graph of the same size and mean degree, ie $C \gg C_{random}$. The network thus has high clustering. Given of size of 680 vertices, small average distance of about 2.6, and the relatively large clustering coefficients which are basically the small-world characteristics[11], the international flight network can be classified as a small-world network.

2.2. Degree distribution

To further investigate the structure of the network, we find the degree distribution of the network. The degree distribution of the network is fitted into exponential, Poisson, linear and power law curves as shown in Table 2.

Types of degree distribution	Equations
Power-law	$P(k) \sim k^{-\gamma}$
Exponential	$P(k) \sim exp(-\frac{k}{m})$, m is a constant
Poisson	$P(k) \sim \frac{e^{-\lambda}\lambda^{k}}{k!}$

Table 2: Different types of degree distribution[12]

It is found that the distribution can be best fitted into a power-law curve. As shown in Fig 1, plotting the graph of lnP against lnk gives a relatively straight line with a slope of $(-\gamma = -1.1043)$. Therefore, the international flight network also behaves like a scale-free network that follows power law.

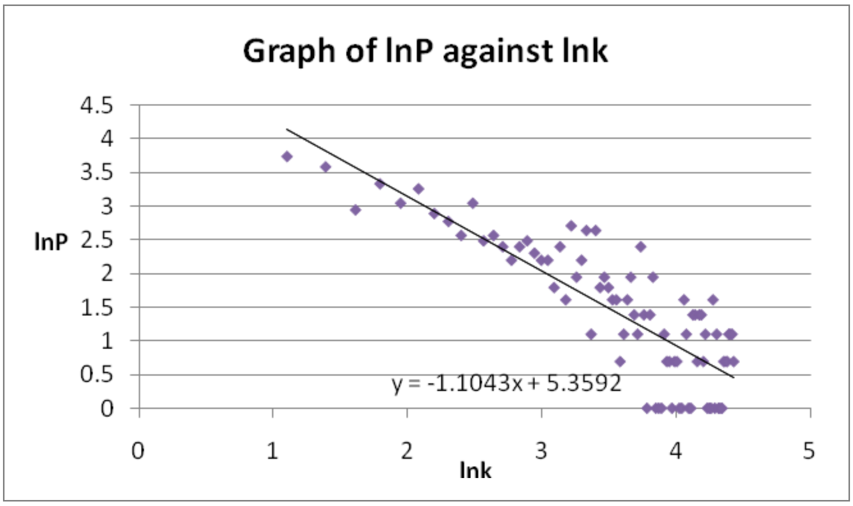

Fig 1: Graph of lnP against lnk

Such scale-free property could be one of the factors that contribute to the small-effect that can be observed in this network. With the existence of 'hubs' with high degree that are connected to minor cities and also other hubs, it helps to minimise the distance between any 2 vertices. These minor cities are mainly the cities with only domestic airports.

2.3. Centrality

Next, we investigate the centrality of the vertices in the international flight network. Table 2 shows the top 10 cities that have the highest centrality for the 3 aspects of centralities: degree, closeness and betweenness.

No.	Degree centrality		Closeness centrality		Betweenness centrality	
	Code	City	Code	City	Code	City
1	366	London	366	London	366	London
2	476	Paris	476	Paris	476	Paris
3	218	Frankfurt	218	Frankfurt	218	Frankfurt
4	35	Amsterdam	35	Amsterdam	445	New York
5	426	Munich	426	Munich	35	Amsterdam
6	530	Rome	530	Rome	190	Dubai
7	392	Manchester	445	New York	51	Atlanta (USA)
8	445	New York	679	Zurich	426	Munich
9	679	Zurich	407	Milan	629	Toronto
10	197	Dusseldorf	190	Dubai	530	Rome

Table 3: Top 10 cities that rank highest for each centrality, namely degree, closeness and betweenness.

As seen in Table 3, the results obtained using three centralities are very similar.

Here we present how a mathematical method can be employed to determine the most alpha city in the world or in other words, a city deemed to be the most important node point in the global economy. Using this method, we measure the socioeconomic positions of the cities from the new perspective of accessibility, represented by the availability of direct flights between cities. Furthermore, the concept of centrality can also be applied in marketing strategy. Knowing the alpha cities, it would help to decide the location of the headquarter of a company so as to maximise profits.

2.4. Network robustness

Network robustness, which is the resilience of the network to failure or removal of nodes, is also an important aspect in the study of network structure. In the international flight network, deletions of nodes can happen due to terrorism, protest in the airports, the influence of weather or even natural disasters such as hurricane. In this project, to investigate the robustness of the network, we systematically remove hubs that are identified in the previous section and observe the change in diameter, mean distance as well as the number of disconnected components after each removal.

Codes of hubs removed	Diameter	Mean distance $\bar{d}$	Number of disconnected components	Size of the largest component
None	8	2.572	NIL	680
366	8	2.645	2	678
476	8	2.606	NIL	679
218	8	2.584	NIL	679
35	8	2.581	NIL	679
426	8	2.575	NIL	679
530	8	2.576	NIL	679
366, 476	8	2.689	2	677
366, 476, 218	8	2.720	2	676
366,476, 218, 35	8	2.740	2	675

Table 4: Details of the removal of hubs and the diameter, mean distance, disconnected components of the resulting networks. The list is arranged according to the centrality of the vertices.

As one hub is removed from the network, the changes made to the network are not very significant as there is only slight change to the distances between vertices (<0.2) while the diameter of the network remains constant. The small change in the average distance could be due to the availability of alternative paths of the same shortest path length that do not pass through the particular vertex to be removed. This is evident in the W matrix as of all pairs of vertices, 77.3% pairs have more than one path in order to travel with the shortest path length. In general, the higher the centrality of the vertex removed, the greater the increase in mean distance. This further indicates the direct relationship between the importance of the cities in the network and their centrality.

Only when more hubs are removed, we can then observe more significant changes such as the disintegration of the network into several disconnected components and greater increase in mean distance. We can therefore conclude that the international flight network is a robust network.

However, in reality, removal of nodes often occurs at random. This can happen to any cities, not necessarily the more important nodes. Therefore, we also observe and analyse how the network changes as vertices are removed at random.

No of vertices removed at random	Diameter	Mean distance $\bar{d}$	Number of disconnected components	Size of the largest component
0	8	2.572	NIL	680
1 (133)	8	2.571	NIL	679
1 (119)	8	2.574	NIL	679
1 (99)	8	2.571	NIL	679
1 (278)	8	2.568	NIL	679
1 (45)	8	2.572	NIL	679
1 (561)	8	2.571	NIL	679
1 (194)	8	2.570	NIL	679
5	8	2.570	NIL	675
34	8	2.565	2	645

Table 5: Details of the removal of vertices at random and the diameter, mean distance, disconnected components of the resulting networks.

As can be seen in Table 5, the random removal of vertices only results in negligible change to the network as compared to that when hubs are removed. Even as 34 vertices (5% of all vertices) are deleted, the network still remains intact as only one city is disconnected from the network. Besides, there is no significant change of mean distance when vertices are removed. This is due to the fact that randomly removed vertices are not important in the structure of the whole network, in stark contrast to the results of the mean distance after removal of important nodes.

The high robustness of the network to random failure could be related to the degree distribution and structure of the network. The power law distribution suggests that most of the vertices are of small degree and they are more likely to be the ones being affected as failures occur randomly.

Therefore, such deletions do not have great impact on the flight network. On the other hand, the network is more fragile to targeted attacks when hubs that are high-degree nodes are removed.

3. CONCLUSION AND FUTURE EXTENSION

The international flight network is proven to be a small world network, due to its small mean vertex-to-vertex distance and high clustering coefficient, which justifies the relative ease of travelling and transportation in this age. Additionally, it is also a scale-free network in which several alpha cities play a far more important role than any other cities. The presence of these cities is further confirmed by data on centralities. That the network still remains intact upon the random removal of 5% of vertices bears testimony to the fact that the network is robust, unlikely to be disturbed by small incidents.

In the light of practicality, the project provides a gauge for multinationals considering setting up a headquarter as the data of centralities is an indication of the extent of accessibility of the city.

However, there are several limitations of the project, among which is the inherent problem with the data collection process, as the website is unable to provide exhaustive data on direct flights. Searching more websites can ameliorate this condition.

Besides the existence of a direct flight between two cities, other considerations, such as the frequency of flights, are needed to determine how accessible and cosmopolitan a certain city is. Also, during business strategising, the actual distance between two cities is absolutely required in the calculation of transportation cost in addition to the accessibility of certain city. This can be improved by using a weighted graph that gives a rating to every edge in the graph.

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5. Appendices

5.1. Appendix A – Cities arranged by continents

Africa

Cities	*Countries*	*Cities*	*Countries*
Abidjan	Ivory Coast	Kilimanjaro	Tanzania
Abu Simbel	Egypt	Lagos	Nigeria
Abuja	Nigeria	Libreville	Gabon
Accra	Ghana	Lilongwe	Malawi
Addis Ababa	Ethiopia	Luanda	Angola
Agadir	Morocco	Lusaka	Zambia
Alexandria Any	Egypt	Luxor	Egypt
Algiers	Algeria	Mahe Island	Seychelles
Antananarivo	Madagascar	Malabo	Equatorial Guinea
Asmara	Eritrea	Marrakech Menara	Morocco
Aswan Daraw	Egypt	Marsa Alam	Egypt
Banjul	Gambia	Mauritius	Mauritius
Benghazi	Libya	Mombasa	Kenya
Cairo	Egypt	Monastir	Tunisia
Cape Town	South Africa	Moroni Prince Said Ibrahim In	Comoros
Casablanca Any	Morocco	Nairobi Any	Kenya
Dakar	Senegal	Ouarzazate	Morocco
Dar Es Salaam	Tanzania	Oujda L. Angades	Morocco
Djerba	Tunisia	Port Harcourt	Nigeria
Djibouti	Djibouti	Praia	Cape Verde
Douala	Cameroon	Rabat Sale	Morocco
Dzaoudzi	Mayotte	Sal	Cape Verde
Entebbe	Uganda	Sharm El Sheikh.	Egypt
Fez Sais	Morocco	St Denis de la Reunion	Réunion
Freetown Lungi International	Sierra Leone	Taba	Egypt
Harare	Zimbabwe	Tangier Boukhalef	Morocco
Hassi Messaoud	Algeria	Tozeur	Tunisia
Hurghada	Egypt	Tripoli	Libya
Johannesburg	South Africa	Tunis Carthage	Tunisia
Kabul	Afghanistan	Windhoek Any	Namibia
Khartoum	Sudan	Zanzibar	Tanzania

North America

Cities	*Countries*	*Cities*	*Countries*
Acapulco	Mexico	Atlantic City International	USA
Albany	USA	Baltimore Balt/Wash	USA
Antigua	Antigua and Barbuda	Belize City Any	Belize
Aruba	Aruba	Bermuda International	Bermuda
Atlanta	USA		
Birmingham Alabama	USA	Holguin	Cuba

Boston Logan International	USA
Bridgetown	Barbados
Buffalo Niagara	USA
Calgary	Canada
Cancun	Mexico
Cayman Brac Is	Cayman Islands
Cayo Coco	Cuba
Charleston South Carolina	USA
Charleston West Virginia	USA
Charlotte Douglas	USA
Chicago Any	USA
Cincinnati Northern Kentucky	USA
Cleveland Hopkins International	USA
Colorado Springs Peterson	USA
Cozumel	Mexico
Dallas Any	USA
Deer Lake	Canada
Denver	USA
Detroit Wayne County	USA
Dominica Cane Field	Dominica
Edmonton International	Canada
Flores	Guatemala
Fort De France	Martinique
Freeport	The Bahamas
George Town	The Bahamas
Governors Harbour	The Bahamas
Grand Cayman Island	Cayman Islands
Greensboro/High Point	USA
Grenada	Grenada
Guatemala City	Guatemala
Halifax International	Canada
Hartford Any	USA
Havana	Cuba
Hilo	USA
Hilton Head	USA

Honolulu International	USA
Houston Any	USA
Huntsville International	USA
Indianapolis	USA
Jacksonville International	USA
Kahului	USA
Kauai Island Lihue	USA
Key West International	USA
Kingston Norman Manley	Jamaica
Kona	USA
La Ceiba	Honduras
La Romana	Dominican Republic
Las Vegas	USA
Little Cayman	Cayman Islands
Long Island	The Bahamas
Los Angeles	USA
Madison	USA
Managua	Nicaragua
Memphis International	USA
Mexico City Juarez International	Mexico
Miami International	USA
Milwaukee General Mitchell	USA
Minneapolis St Paul	USA
Moncton	Canada
Montego Bay	Jamaica
Montreal Any	Canada
Montrose	USA
Myrtle Beach AFB	USA
Nashville	USA
Nassau International	The Bahamas
Nevis	St Kitts And Nevis
New Orleans Louis Armstrong	USA
New York Any	USA
Norfolk International	USA
San Pedro Sula	Honduras

North Eleuthera	The Bahamas	San Salvador	El Salvador
Orlando Any	USA	Santa Clara	Cuba
Ottawa International	Canada	Santo Domingo Las Americas	Dominican Republic
Philadelphia International	USA	Sarajevo International	Bosnia And Herzegovina
Phoenix	USA	Savannah/Hilton Head	USA
Pittsburgh Int'l Apt.	USA	Seattle Any	USA
Pointe-a-Pitre	Guadeloupe	St Barthelemy	St Barths
Port Au Prince	Haiti	St John's	Canada
Portland	USA	St Kitts	St Kitts And Nevis
Providence	USA	St Louis	USA
Puerto Plata	Dominican Republic	St Lucia Any	St Lucia
Puerto Vallarta	Mexico	St Lucia Vigie	St Lucia
Punta Cana	Dominican Republic	St Maarten	Netherlands Antilles
Quebec Airport	Canada	Tampa International	USA
Raleigh/Durham	USA	Tegucigalpa	Honduras
Saba Island	Netherlands Antilles	Toronto Any	Canada
Salt Lake City	USA	Vancouver	Canada
San Antonio International	USA	Varadero	Cuba
San Diego Any	USA	Washington Any	USA
San Francisco International	USA	West Palm Beach International	USA
San Jose Cabo	Mexico	Wichita Mid-Continent	USA
San Jose Juan Santamaria	Costa Rica	Winnipeg	Canada
San Juan Luis Munoz Marin	Puerto Rico		

South America

Cities	*Countries*	*Cities*	*Countries*
Asuncion	Paraguay	Montserrat	Montserrat
Bogota	Colombia	Natal	Brazil
Brasilia	Brazil	North Caicos	Turks And Caicos
Buenos Aires	Argentina	Porlamar	Venezuela
Caracas	Venezuela	Providenciales	Turks And Caicos
Cartagena	Colombia	Rio De Janeiro	Brazil
Cayenne	French Guiana	Salt Cay	Turks And Caicos
Grand Turk Is	Turks And Caicos	Salvador	Brazil
Guayaquil	Ecuador	Sao Paulo	Brazil
Lima	Peru	South Caicos	Turks And Caicos
Tobago	Trinidad And Tobago	Trinidad	Trinidad And Tobago

Asia

Cities	*Countries*	*Cities*	*Countries*
Abha	Saudi Arabia	Delhi	India
Abu Dhabi	UAE	Dhaka	Bangladesh
Aden	Yemen	Doha Airport	Qatar
Agatti Island	India	Dubai	UAE
Agra	India	Durban International	South Africa
Ahmedabad	India	Ekaterinburg	Russia
Aktau	Kazakhstan	Ercan	Turkey
Aleppo	Syria	Fukuoka	Japan
Almaty	Kazakhstan	Gaziantep	Turkey
Amman any	Jordan	General Santos	Philippines
Amritsar	India	Goa	India
Ankara (Any)	Turkey	Guangzhou	China
Antalya Havalimani	Turkey	Hanoi	Vietnam
Aqaba	Jordan	Ho Chi Minh City	Vietnam
Ashgabat	Turkmenistan	Hong Kong	Hong Kong
Bahrain	Bahrain	Hyderabad	India
Baku Heydar Aliyev International	Azerbaijan	Islamabad	Pakistan
Balikpapan	Indonesia	Istanbul Any	Turkey
Bandar Seri Begawan	Brunei	Izmir	Turkey
Bangalore	India	Jaipur	India
Bangkok	Thailand	Jakarta Soekarno-Hatta	Indonesia
Bario	Malaysia	Jeddah	Saudi Arabia
Beijing	China	Jodhpur	India
Beirut	Lebanon	Johor Bahru	Malaysia
Bintulu	Malaysia	Kagoshima	Japan
Bishkek	Kyrgyzstan	Kaliningrad	Russia
Bodrum	Turkey	Kaohsiung	Taiwan
Busan	South Korea	Karachi	Pakistan
Calicut	India	Kathmandu	Nepal
Cebu	Philippines	Kobe	Japan
Chandigarh	India	Kochi	India
Chengdu	China	Koh Samui	Thailand
Chennai	India	Kolkata	India
Chiang Mai	Thailand	Kota Bharu	Malaysia
Chittagong	Bangladesh	Kota Kinabalu	Malaysia
Chongqing	China	Krabi	Thailand
Colombo Bandaranayake	Sri Lanka	Kuala Lumpur	Malaysia
Cox's Bazar	Bangladesh	Kuala Terengganu	Malaysia
Dalaman	Turkey	Kuantan	Malaysia
Damascus	Syria	Kuching	Malaysia
Dammam	Saudi Arabia	Kuwait	Kuwait
Davao	Philippines	Labuan	Malaysia
Lahore	Pakistan	Nagasaki	Japan
Langkawi	Malaysia	Sandakan	Malaysia
Larnaca	Cyprus	Sanya City	China
Lawas	Malaysia	Sapporo Any	Japan

Lucknow	India	Seoul	South Korea
Madinah	Saudi Arabia	Shanghai	China
Male	Maldives	Shenzhen	China
Manila	Philippines	Sibu	Malaysia
Marudi	Malaysia	Siem Reap	Cambodia
Mataram	Indonesia	Simla	India
Matsuyama	Japan	Singapore	Singapore
Miri	Malaysia	Surabaya	Indonesia
Mulu	Malaysia	Sylhet	Bangladesh
Mumbai	India	Taipei Any	Taiwan
Muscat	Oman	Tashkent	Uzbekistan
Mysore	India	Tawau	Malaysia
Nagoya Any	Japan	Tbilisi	Georgia
Nejran	Saudi Arabia	Tehran	Iran
Niigata	Japan	Tel Aviv	Israel
Okinawa Naha	Japan	Thiruvananthapuram	India
Osaka Any	Japan	Tokyo Any	Japan
Ovda	Israel	Udaipur	India
Paphos	Cyprus	Uralsk	Russia
Penang	Malaysia	Urumqi	China
Peshawar	Pakistan	Varanasi	India
Phnom Penh	Cambodia	Vientiane	Laos
Phuket	Thailand	Xiamen	China
Pune	India	Yangon	Myanmar
Riyadh	Saudi Arabia	Yerevan	Armenia
Sana'a	Yemen		

Europe

Cities	*Countries*	*Cities*	*Countries*
Aalborg	Denmark	Balaton	Hungary
Aalesund Vigra	Norway	Barcelona Any	Spain
Aarhus	Denmark	Bari	Italy
Aberdeen	United Kingdom	Barra	United Kingdom
Akureyri	Iceland	Basel Mulhouse Freiburg	Switzerland
Albacete	Spain	Bastia-Corsica	France
Alderney	United Kingdom	Belfast Any	United Kingdom
Alghero Sardinia	Italy	Belgrade	Serbia
Alicante	Spain	Benbecula	United Kingdom
Almeria	Spain	Bergen	Norway
Amsterdam	Netherlands	Bergerac	France
Ancona	Italy	Berlin Any	Germany
Anglesey	United Kingdom	Bern	Switzerland
Antwerp Deurne	Belgium	Beziers	France
Asturias	Spain	Biarritz	France
Athens	Greece	Bilbao	Spain
Avignon	France	Billund	Denmark
Baden-Baden	Germany	Birmingham	United Kingdom
Blackpool	United Kingdom	Edinburgh	United Kingdom
Bodo	Norway	Esbjerg	Denmark
Bologna Any	Italy	Evenes	Norway
Bordeaux	France	Exeter	United Kingdom
Bournemouth	United Kingdom	Fagernes Valdres	Norway

Bratislava	Slovakia	Faro	Portugal
Bremen	Germany	Figari Sud Corse	France
Brest	France	Florence	Italy
Brindisi	Italy	Floro	Norway
Bristol	United Kingdom	Frankfurt Any	Germany
Brno- Turany	Czech Republic	Friedrichshafen	Germany
Brussels Any	Belgium	Fuerteventura	Spain
Bucharest Any	Romania	Galway	Ireland
Budapest	Hungary	Gdansk	Poland
Burgas	Bulgaria	Geneva	Switzerland
Bydgoszcz	Poland	Genoa	Italy
Cagliari	Italy	Gibraltar	Gibraltar
Calvi	France	Glasgow Any	United Kingdom
Campbeltown	United Kingdom	Gloucestershire	United Kingdom
Carcassonne	France	Gothenburg Any	Sweden
Cardiff	United Kingdom	Granada	Spain
Catania Fontanarossa	Italy	Gran Canaria Las Palmas	Spain
Chambery	France	Graz	Austria
Chisinau	Moldova	Grenoble	France
Ciudad Real	Spain	Groningen	Netherlands
Cluj- Napoca	Romania	Guernsey	United Kingdom
Cologne	Germany	Hamburg Any	Germany
Constanta Kogalniceanu	Romania	Helsingborg Angelholm	Sweden
Copenhagen	Denmark	Haugesund	Norway
Corfu	Greece	Hannover	German
Cork	Ireland	Helsinki Vantaa	Finland
Crete Any	Greece	Humberside	United Kingdom
Cuneo Levaldigi	Italy	Iasi	Romania
Deauville St. Gatien	France	Ibiza	Spain
Derry	United Kingdom	Innsbruck	Austria
Dinard	France	Inverness	United Kingdom
Doncaster Sheffield	United Kingdom	Isafjordur	Iceland
Donegal	Ireland	Islay Glenegedale	United Kingdom
Dortmund	Germany	Isle Of Man	United Kingdom
Dresden	Germany	Isles Of Scilly Any	United Kingdom
Dublin	Ireland	Jerez	Spain
Dubrovnik	Croatia	Jersey	United Kingdom
Dundee	United Kingdom	Kalamata Interntional	Greece
Durham Tees Valley	United Kingdom	Kardla	Estonia
Dusseldorf Any	Germany	Karlovy Vary	Czech Republic
East Midlands	United Kingdom	Katowice	Poland
Eday	United Kingdom	Kaunas	Lithuania
Kefallinia	Greece	Menorca	Spain
Kent International Airport	United Kingdom	Milan Any	Italy
Kerry	Ireland	Minsk International	Belarus
Kiev	Ukraine	Monaco	Monaco
Klagenfurt	Austria	Montpellier	France
Knock	Ireland	Moscow Any	Russia
Kos	Greece	Mulhouse	France

Kosice	Slovakia
Krakow	Poland
Kristiansand Kjevik	Norway
Kristiansund	Norway
Kuressaare	Estonia
La Coruna	Spain
La Palma	Spain
La Rochelle	France
Lamezia Terme	Italy
Lands End	United Kingdom
Lanzarote	Spain
Le Touquet Paris Plag	France
Leeds Bradford	United Kingdom
Leipzig Any	Germany
Lemnos	Greece
Lille Any	France
Limoges	France
Linz	Austria
Lisbon	Portugal
Liverpool	United Kingdom
Ljubljana	Slovenia
Lodz	Poland
London Any	United Kingdom
Lorient	France
Lourdes	France
Lugano	Switzerland
Lulea Kallax	Sweden
Luqa Malta International	Malta
Luxembourg City	Luxembourg
Lviv	Ukraine
Lydd	United Kingdom
Lyon	France
Madeira	Portugal
Madrid	Spain
Malaga	Spain
Malmo Sturup	Sweden
Manchester	United Kingdom
Marseille	France
Memmingen Allgäu	Germany
Porto	Portugal
Poznan	Poland
Prague	Czech Republic
Preveza	Greece
Pristina	Serbia
Pula	Croatia
Quimper	France
Rennes	France
Reykjavik Any	Iceland
Munich	Germany
Munster Osnabruck	Germany
Murcia	Spain
Mykonos	Greece
Mytilene	Greece
Nantes	France
Naples International	Italy
Newcastle	United Kingdom
Newquay	United Kingdom
Nice	France
Nimes	France
North Ronaldsay	United Kingdom
Norwich	United Kingdom
Nuremberg	Germany
Olbia	Italy
Orkney	United Kingdom
Oslo Any	Norway
Ostersund Froesoe	Sweden
Oulu	Finland
Paderborn	Germany
Palanga International	Lithuania
Palermo	Italy
Palma-Majorca	Spain
Pamplona	Spain
Papa Westray	United Kingdom
Paris Any	France
Parma	Italy
Pau	France
Penzance	United Kingdom
Perpignan	France
Perugia Santegidio	Italy
Pescara	Italy
Pisa	Italy
Plymouth	United Kingdom
Podgorica	Montenegro
Poitiers	France
Ponta Delgada	Portugal
Poprad-Tatry	Slovakia
Port Blair	India
Stuttgart	Germany
Sumburgh Shetlands	United Kingdom
Szczecin Goleniow	Poland
Tallinn	Estonia
Tampere	Finland
Tartu	Estonia
Tenerife Any	Spain
Thessaloniki	Greece
Timisoara	Romania

Rhodes	Greece	Tirana	Albania
Riga International	Latvia	Tiree	United Kingdom
Rijeka	Croatia	Tirgu Mures	Romania
Rimini	Italy	Tivat	Montenegro
Rodez	France	Toulon	France
Rome Any	Italy	Toulouse	France
Rotterdam	Netherlands	Tours	France
Rovaniemi	Finland	Trapani Birgi	Italy
Rzeszow	Poland	Trieste	Italy
Salzburg	Austria	Tromso	Norway
Samos	Greece	Trondheim	Norway
San Sebastian Fuenteraba	Spain	Turin	Italy
Sanday	United Kingdom	Umeå	Sweden
Santander	Spain	Valencia	Spain
Santiago de Compostela	Spain	Valladolid	Spain
Santorini (Thira)	Greece	Varna	Bulgaria
Seville	Spain	Venice Any	Italy
Shannon	Ireland	Verona Any	Italy
Skiathos Island National	Greece	Vienna Any	Austria
Skopje	Macedonia, Republic of	Vigo	Spain
Sligo	Ireland	Vilnius	Lithuania
Sofia	Bulgaria	Vitoria	Spain
Sonderborg	Denmark	Volos	Greece
Sorvagur	Faroe Islands	Warsaw	Poland
Southampton	United Kingdom	Waterford	Ireland
Split	Croatia	Westray	United Kingdom
St Etienne	France	Wick	United Kingdom
St Gallen Altenrhein	Switzerland	Wroclaw	Poland
St Petersburg Pulkovo	Russia	Zadar	Croatia
Stavanger	Norway	Zagreb	Croatia
Stockholm Any	Sweden	Zakinthos	Greece
Stornoway	United Kingdom	Zaragoza	Spain
Strasbourg	France	Zurich	Switzerland
Stronsay	United Kingdom	Zweibrucken	Germany

Oceania and Australia

Cities	*Countries*	*Cities*	*Countries*
Adelaide	Australia	Dunedin	New Zealand
Ajaccio	France	Eindhoven	Netherlands
Alice Springs	Australia	Hobart	Australia
Auckland International	New Zealand	Melbourne	Australia
Ayers Rock	Australia	Perth	Australia
Brisbane	Australia	Queenstown	New Zealand
Cairns	Australia	Rotorua	New Zealand
Canberra	Australia	Sydney	Australia
Christchurch	New Zealand	Wellington	New Zealand
Darwin	Australia		

5.2. Appendix B – MATLAB codes

5.2.1. Breadth First Search (BFS)

- **Obtain distance matrix, D from incidence matrix, a**

```
D=[]
for i=1:size(a,1)
    count=1;
    c1=a(i,:)
    d=zeros(1,size(a));
    d1=zeros(1,size(a));
    d2=zeros(1,size(a));
    g=1:size(a,2)
    d=d+[zeros(1,i-1) 1 zeros(1,size(a,2)-i)]
    count=count+1
    for j=1:size(a,2)
        if c1(1,j)==1
            d1(1,j)=d1(1,j)+count
        end
    end
    d=d+d1
    while count<8
        f=d==count
        h=zeros(1,size(a))
        h(f)=h(f)+g(f)
        count=count+1
        for i=h
            for j=1:size(a,2)
                if i>0
                    if a(i,j)==1
                        c2=a(i,:)
                        d2(1,j)=d2(1,j)+count
                        e=d==0
                        d(e)=d(e)+d2(e)
                    end
                else
                    continue;
                end
            end
            continue;
        end
    end
    d=d-1
    D=[D;d]
end
```

5.2.2. Obtain a matrix W from distance matrix, D and incidence matrix, a, where the (i,j) entry of W is the number of different $v_i – v_j$ walks of minimum length.

```
W=[]
for i=1:size(D)
    count=1
    v1=D(i,:)
    y=1:size(D)
    u=v1==1
    while count<size(D)-2
        count=count+1
        x2=v1==count
        v=zeros(1,size(D))
        v(x2)=v(x2)+y(x2)
        for i=v
            if i>0
```

```
                w1=a(i,:)
                w2=v1==(count-1)
                w=w1+w2==2
                w3=zeros(1,size(D))
                w3(w)=w3(w)+u(w)
                w4=sum(w3)
                x3=[zeros(1,i-1) 1 zeros(1,size(a,2)-i)]
                u=u+x3*w4
            else
                continue;
            end
        end
    end
    W=[W;u]
end
```

5.2.3. Degree centrality of each vertex, DC

```
DC=sum(a)/(size(a,1)-1)
```

5.2.4. Closeness centrality of each vertex, CC

```
CC=(size(D,1)-1)./sum(D)
```

5.2.5. Betweenness centrality of each vertex, B

```
BC=[]
for j=1:size(D)
    z1=[]
    for i=1:size(D)
        if i==j
            u1=zeros(1,size(D))
            z1=[z1;u1]
        else
            s=D(i,j)
            t=W(i,j)
            v1=D(i,:)
            y=1:size(D)
            u1=[zeros(1,j-1) t zeros(1,size(D)-j)]
            while s<size(D)-2
                s=s+1
                x2=v1==s
                v=zeros(1,size(D))
                v(x2)=v(x2)+y(x2)
                for i=v
                    if i>0
                        w1=a(i,:)
                        w2=v1==(s-1)
                        w3=u1>0
                        w=w1+w2+w3==3
                        w4=zeros(1,size(D))
                        w4(w)=w4(w)+u1(w)
                        w5=sum(w4)
                        x3=[zeros(1,i-1) 1 zeros(1,size(a,2)-i)]
                        u1=u1+x3*w5
                    else
                        continue;
                    end
                end
            end
            z1=[z1;u1]
        end
    end
```

```
        z1(:,j)=0
        z2=z
        z2(:,j)=0
        z2(j,:)=0
        b=sum(sum(z1))/sum(sum(z2))
        BC=[BC b]
    end
```

7.2.6. Global clustering coefficient of the network

- obtained from matrices a, D and z.

```
cl=[]
g=1:size(a,2)
for i=1:size(a)
    h=zeros(1,size(a))
    b=zeros(1,size(a))
    d=a(i,:)
    f=d==1
    h(f)=h(f)+g(f)
    for i=h
        if i>0
            t=a(i,:)
            b(f)=b(f)+t(f)
        else
            continue
        end
    end
    m=sum(b)
    cl=[cl m]
end
k=zeros(size(a))
n=D==2
k(n)=k(n)+z(n)
C=sum(cl)/sum(sum(k))
```

7.2.8. Network average clustering coefficient

- obtained from matrix a.

```
cl=[]
g=1:size(a,2)
for i=1:size(a)
    h=zeros(1,size(a))
    b=zeros(1,size(a))
    d=a(i,:)
    k=sum(d)
    f=d==1
    h(f)=h(f)+g(f)
    for i=h
        if i>0
            t=a(i,:)
            b(f)=b(f)+t(f)
        else
            continue
        end
    end
    m=sum(b)
    n=m/(k^2-k)
    cl=[cl n]
end
x=zeros(1,size(a))
y=cl<=1
x(y)=x(y)+cl(y)
C=sum(C1)/sum(sum(k))
```