

# Weak Roman Domination

## Abstract

This project focuses on the research of weak Roman domination number. Weak Roman domination is a concept proposed in 2002, which is a new strategy in response to the famous Roman domination problem. In graph theoretic terminology, let  $G=(V,E)$  be a graph and  $f$  be a function  $f:V \rightarrow \{0,1,2\}$ ; a vertex  $u$  with  $f(u)=0$  is said to be undefended with respect to  $f$  if it is not adjacent to a vertex with positive weight. The function  $f$  is a weak Roman dominating function (WRDF) if each vertex  $u$  with  $f(u)=0$  is adjacent to a vertex  $v$  with  $f(v)>0$  such that the function  $f':V \rightarrow \{0,1,2\}$  with  $f'(u)=1, f'(v)=f(v)-1, f'(w)=f(w)$  if  $w \in V - \{u,v\}$  has no undefined vertex. The weight of  $f$  is  $w(f) = \sum_{v \in V} f(v)$ . The Weak Roman Domination number, denoted by  $\gamma_r(G)$ , is the minimum weight of a WRDF in  $G$ .

Successful approaches have been made to show that Weak Roman dominating index  $r(xy)$  is between 0 and 1, with all the possible cases fully discussed. Furthermore, the paper shows  $\gamma_r(G) \leq \left\lfloor \frac{2n}{3} \right\rfloor$  for any connected graph of order  $n$  and  $\gamma_r(G) \leq \left\lfloor \frac{2n-2}{3} \right\rfloor$  for unicyclic graphs. Those upper bounds are crucial when applying the theory into real-life scenarios. This paper also characterizes graphs for those upper bounds to be achieved. For further research, a comparison between Roman Domination, Weak Roman Domination and general domination can be made. Besides, more research on various types of graphs and real-life application can also be done.