Science and Mathematics Talent Programme (SMTP)

Research Paper

(Chinese) Chess Titans:
A Mathematical Approach

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CONTENTS

1. Abstract

There has been great interest in generating computer programs to play Xiangqi (Chinese Chess) after success in international chess. This project aims to work out the factors that affect the middle game of Xiangqi and use these factors to model an indicative function that computes the advantage of a certain player at a certain point in time, in other words, provides a probabilistic indication of whether a player will defeat his opponent judging from that particular situation. So far, the results have been conclusive and an indicative function that is mathematically strategised have been found. An Excel file has also been computed based on the indicative function, allowing a user to interact with the Excel file to find out his best move from the current situation.

2. Introduction

DEEP BLUE's victory over World Chess Champion Kasparov in 1997 took many chess players by surprise. Following this, many developers began work on the corresponding Oriental variant: Chinese Chess (Xiangqi). In 2006, and subsequently in 2007, a match of a similar scale took place between the top Xiangqi player, 许银川 and the top Xiangqi program, 棋天大圣. Two games were played, with a draw in both. This triggered a sensational urgency for human players and computer program developers to work harder. A super-program was expected to be developed by year 2010 but it did not come into perceptible existence.

1.1 Rules & Instructions

Chinese Chess is played on a 9x10 board with a "river" in the middle and a "palace" on both sides. Certain pieces cannot cross the "river" and certain pieces cannot come out of the "palace". The objective of the game is to capture the King, known as checkmate, or put the opponent in a situation where he or she is not in check but is left with no legal moves, known as stalemate. There is a detailed description of the rules of the game attached in the appendix.

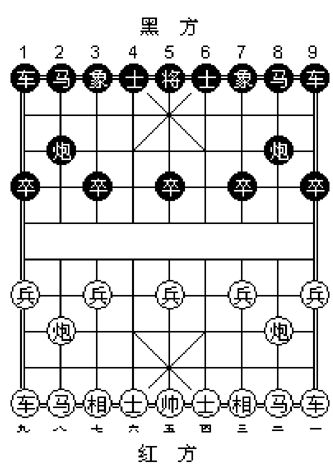


Figure 1

1.2 Terminologies

1. Zero-Sum Game

   A mathematical representation of a situation in which a participant's gain or loss is exactly balanced by the losses or gains of the other participant(s). In the context of Chinese Chess,

$$A_R + A_B = 0$$

   where A_R refers to the advantage of Red and A_B refers to the advantage of Black or the negative disadvantage of Black and vice versa.

2. Indicative Function

   The indicative function is defined on the basis that Xiangqi is a zero-sum game. The indicative function of any instantaneous situation is a numerical value. A positive value refers to an advantage in Red whereas a negative value refers to a disadvantage in Red which means an advantage in Black. The indicative function is used in many algorithms to compare future possible scenarios and decide upon the best move.

3. Combinatorial Game Theory

   Combinatorial Game Theory refers to a branch of Game Theory which studies the strategies and mathematics of two-player games of perfect knowledge such as chess or go An important distinction between this subject and classical game theory is that game players are assumed to move in sequence rather than simultaneously, so there is no randomisation or other information-hiding strategies.

2. Rationale

The choice of Xiangqi in place of other mind games and intellectual games as the game-tree complexity of Xiangqi is next in line following after International Chess. This means that research in Xiangqi is paramount for better understanding in Artificial Intelligence (AI) and Combinatorial Game Theory (CGT).

Game	**State-space complexity***	**Game-tree complexity***
Chess	50	123
Chinese Chess (Xiangqi)	48	150
Shogi	71	226
Go	160	400

Table 1: State-space complexity and game-tree complexity given by the power of 10.
**Both indicators of complexity are measurements of the board size and variations in the game respectively.*

A Chinese Chess Game typically consists of three main parts – opening game, middle game and endgame, which is approximately the 1^{st} -12^{th} moves, 13^{th} -25^{th} moves and 26^{th} -50^{th} moves respectively. Since there has been thorough research done on opening book and equal amount of effort put into end game databases, this project would like to focus mainly on the middle game, which is also the most complex and complicated section of the whole game.

The most important reason to identify the indicative function as the basis of our project is that current ways of computing the indicative function, though effective, they are conventional and not mathematically based. They are based on experimental trials and master games which are still bound to contain a certain degree of error, though small. This means that *i(x)* is not 100% accurate and improvements can be made. Moreover, the current *i(x)* may be accurate only for most cases. In extreme cases, such as sacrifice, *i(x)* has a high chance of a paradox, leading to it being very inaccurate.

Over and above all, our project possesses potential commercial values as it offers an authentic method for Chess players to further hone their Chess skills.

3. Aims & Objectives

Our project aims to create an algorithm to play Chinese Chess well in middle game based on mathematical strategies through the following steps:

1. Determine the factors that affect the strategies of the middle game
2. Mathematically model the different factors individually and observe the extent at which they affect middle game
3. Determine how does the different factors culminate into an effective strategy
4. Use mathematics to code a program which strategises for the best move

4. Research Problems

This project serves to investigate the following:

1. Calculation of material and positional values (of pieces),
2. Calculation of flexibility (of pieces),
3. Mathematical formula(e) relating the factors affecting *i(x)* together and
4. Mathematically compute the relationship (between pieces)

5. Field of Mathematics

1. Statistics
2. (Combinatorial) Game Theory
3. Probability

6. Scope of Research

Our research would cover the generation of the instantaneous value of the indicative function but will not cover the computational part on the comparison of these values. There are many factors affecting the indicative function but only the three significant factors and factors discovered in the period of our project will be reviewed and discussed in the next section.

7. Literature Review

In this section, the different factors which contribute to the indicative function, *i(x)* (审局函数) will be reviewed. The instantaneous value obtained from the indicative function is essential for our algorithm to strategise the best move given a certain situation. An overall review will be done at the end of the section to discuss the significance of the different factors in our project.

Huang, Xu and Du (2007), in the chapter Artificial Intelligence of their book, on the subject of situational analysis of the game, concluded that there are four different factors which affect *i(x)* in disproportionate percentage - sum of material of pieces, position of pieces, flexibility of pieces, protection and threat of the pieces. The indicative function of an instantaneous situation is a numerical value whereby a positive value constitutes an advantage in Red and a negative value constitutes an advantage in Black (Huang, 2008). *i(x)* is a function of the four aforementioned variables with the value of the piece in each variable being numerically assigned to, in accordance to its relative importance in the game. (Ong, Quek, Tan and Tay, 2007)

7.1 Material

Material refers to a quantitative index of the ability of a piece, which remains constant throughout the game

A material advantage usually leads to a win, especially when the opponent does not have an absolute positional advantage (Fleischer and Khan, 2002). Huang *et.al.* wrote that the material aspect of *i(x)* is not a variable but a constant which is the sum of the material, with a value assigned to it and remain constant throughout the game even as the situation varies.

Since a player immediately loses the game once his King is captured, the value assigned to the King is substantially larger than the other pieces (Huang *et.al.*, 2007). However, the value assigned to the King cannot be infinite as no significance will be placed on other pieces leading to the loss of these pieces which makes it easier for one to lose the game. At the initial phase of our project, the table of values given by象棋大全is adopted. However, the project aims to mathematically investigate these relationships and attain our own table of values.

King	Chariot	Horse	Cannon	Elephant	Advisor	Pawn
8000	1000	450	470	160	170	60

Table 2: Material values of pieces

7.2 Position

Huang (2008) reported that pieces are of higher value at strategic positions as the effective utility of the pieces increase, especially applicable to pieces whose next move can reach the palace, compromising the safety of the king. Table 3 shows the assigned positional value of a particular piece at various points of the chess board.

14	14	12	18	16	18	12	14	14
16	20	18	24	26	24	18	20	16
12	12	12	18	18	18	12	12	12
12	18	16	22	22	22	16	18	12
12	14	12	18	18	18	12	14	12
12	16	14	20	20	20	14	16	12
6	10	8	14	14	14	8	10	6
4	8	6	14	12	14	6	8	4
8	4	8	16	8	16	8	4	8
-2	10	6	14	12	14	6	10	-2

Table 3.1: Positional values for Chariot

4	8	16	12	4	12	16	8	4
4	10	28	16	8	16	28	10	4
12	14	16	20	18	20	16	14	12
8	24	18	24	20	24	18	24	8
6	16	14	18	16	18	14	16	6
4	12	16	14	12	14	16	12	4
2	6	8	6	10	6	8	6	2
4	2	8	8	4	8	8	2	4
0	2	4	4	-2	4	4	2	0
0	-4	0	0	0	0	0	-4	0

Table 3.2: Positional values for Horse

6	4	0	-10	-12	-10	0	4	6
2	2	0	-4	-14	-4	0	2	2
2	2	0	-10	-8	-10	0	2	2
0	0	-2	4	10	4	-2	0	0
0	0	0	2	8	2	0	0	0
-2	0	4	2	6	2	4	0	-2
0	0	0	2	4	2	0	0	0
4	0	8	6	10	6	8	0	4
0	2	4	6	6	6	4	2	0
0	0	2	6	6	6	2	0	0

Table 3.3: Positional values for Cannon

0	3	6	9	12	9	6	3	0
18	36	56	80	120	80	56	36	18
14	26	42	60	80	60	42	26	14
10	20	30	34	40	34	30	20	10
6	12	18	18	20	18	18	12	6
2	**0**	8	**0**	8	**0**	8	**0**	2
0	**0**	-2	**0**	4	**0**	-2	**0**	**0**
0	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
0	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
0	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**

Table 3.4: Positional values for Pawn

Values of pawn in bold implies that such positions are non-existential

7.3 Flexibility

Apart from material or positional value, the environment of the pieces, i.e. flexibility must also be considered. Flexibility, although extremely difficult to compute, is extremely vital in the chess board, especially to Horses and to a smaller extent, Elephants. An advantage in flexibility of a player over his opponent allows him to control the situation and broaden his strategic advantage (Huang, 2008).

In COMPUTER CHINESE CHESS, Yen, Chen, Yang and Hsu came to a consensus that calculating the relative flexibility of a piece in the game would cost too much time and only flexibility of the Horses should considered. Even though the eventual contribution of these factors to the indicative function is small, it is included to further increase accuracy of our algorithm as our focus is more on mathematics rather than efficacy.

7.4 Evaluation

The above three discussed factors are instrumental and vital in calculating an accurate indicative function. In the context of material and position of pieces, the values obtained from our sources are highly unreliable as they are only based on experimental trials and not based on mathematical calculations. Hence, it is our priority to mathematically self-tabulate our own set. Flexibility is significantly more complex than the above two functions and hence the function is split into dynamic flexibility, which changes throughout the game and the second component which is to be included in the material function since it will remain constant throughout the game and directly determines the ability of a piece, the probability in which a piece is unrestricted. Threat and protection of a piece does not contribute as an individual variable but instead is a subset of the relationship between pieces.

8. Methodology

In order to achieve our objectives, we must have an outline of the methods that we are using.

Firstly, we will need to read up on books relating to common middle game strategies and the fields of mathematics involved, so as to gain theoretical knowledge to apply to our Research Problems.

Secondly, we will start by observing the behavior of the different pieces in the game and their relative importance in terms of material first. The effective utility of the different pieces will be mathematically calculated using probability for the flexibility aspect and we will attempt a mathematical deciphering of the theoretical approach in the positional aspect. The positional values will be computed using Microsoft Excel using a common formula.

Thirdly, we need to draw linkages between different factors which affect the indicative function. Hence, we can express the indicative function, mathematically as a culmination of the three variables, which are sum of material of pieces, position of pieces and the flexibility of pieces.

Fourthly, we can find mathematical relations between different pieces by a similar method but this time we need to consider both your pieces and your opponent's.

Finally, if time permits, we can create another excel file to compute the indicative function and subsequently an algorithm. To test the strength of the algorithm and the accuracy of our formula, we can play the algorithm against a top level player.

This methodology hence allows us to complete the project and fulfill all the objectives.

9. Findings

In this section, our findings for the mathematical formulation of all the factors affecting the indicative function, with which there are no double computation of values are presented. The factors affecting the indicative function are material, position and flexibility. Quantitisation of the relationship between pieces has also been completed.

9.1 Material

Material refers to a quantitative index of the ability of a piece, which remains constant throughout the game. The material function can be generally formulated as:

$$M(x) = \left[\sqrt{\max(moves) \times \max(captures)} \right] \times P(unrestricted)$$

since the maximum number of possible moves and the maximum number of possible captures are referring to the same variable but different ways to measure this variable, a square root is utilized. For convenience, the following notations are assigned.

King	Chariot	Horse	Cannon	Elephant	Advisor	Pawn
$M(k)$	$M(r)$	$M(h)$	$M(c)$	$M(e)$	$M(a)$	$M(p)$

Table 4: Notations of pieces

At this point of time, it is important to note that the $P(unrestricted)$ is the same for $M(r)$, $M(h)$ and $M(c)$. It is also important to note that $M(e)$ and $M(a)$ is computed using the average of the probability of the occurrence of a certain position as the piece takes on different $P(unrestricted)$ at different positions unlike the attacking pieces. More importantly, $M(p)$ cannot be individually calculated but must be broken down into two components $M(p_b)$ and $M(p_a)$, which refers to the material functions of the pawns before and after crossing the river respectively. To determine $P(unrestricted)$, the probability in which a piece is restricted which equals $1 - P(unrestricted)$ can first be determined.

$$
\begin{aligned}
P(restricted) &= \frac{85! \div (85-26)!}{89! \div (89-26)!} \\
&= \frac{85! \div 59!}{89! \div 63!} \\
&= \frac{85!}{59!} \times \frac{63!}{89!} \\
&= \frac{63 \times 62 \times 62 \times 60}{89 \times 88 \times 87 \times 86} \\
&= 0.756037575
\end{aligned}
$$

$$\therefore P(unrestricted) = 1 - P(restricted) = 0.243962425$$

It follows that:

$$M(r) = \sqrt{17 \times 17} \times 0.24396 = 4.147$$

$$M(h) = \sqrt{8 \times 8} \times 0.24396 = 1.952$$

$$M(c) = \sqrt{17 \times 4} \times 0.24396 = 2.012$$

However, the probability of being unrestricted for an elephant which is not at the central position differs and hence is denoted as $P(unrestricted_0)$. Similarly, it can be computed as:

$$
\begin{aligned}
1 - P(unrestricted_0) &= \frac{87 \times 84 \times 83 \times \ldots \times 62}{89 \times 88 \times 87 \times \ldots \times 64} \\
&= \frac{63 \times 62}{89 \times 88} \\
&= 0.498723187
\end{aligned}
$$

$$\therefore P(unrestricted_0) = 0.501276813$$

$$\therefore M(e) = \frac{6M(e_s) + M(e_c)}{7} = \frac{6(\sqrt{2 \times 2} \times 0.50128) + \sqrt{4 \times 4} \times 0.24396}{7} = 0.9987$$

For $M(a)$, $M(p_b)$ and $M(p_a)$, the probability of it being unrestricted is 1, since each piece can only move one step per move, regardless of capture. It is of a similar concept of the horse but the horse can be restricted by other means. It also follows that:

$$M(a) = \frac{4M(a_s) + M(a_c)}{5} = \frac{4(\sqrt{1\times1}\times1) + \sqrt{4\times4}\times1}{5} = 1.600$$

$$M(p_b) = \sqrt{1\times1}\times1 = 1$$

$$M(p_a) = \frac{2(\sqrt{2\times2}\times1) + 3(\sqrt{3\times3}\times1)}{5} = 2.600$$

Since a player immediately loses the game once his King is captured, the value assigned to the King is substantially larger than the other pieces (Huang *et.al.*, 2007). However, the value assigned to the King cannot be infinite as no significance will be placed on other pieces leading to the loss of these pieces which makes it easier for one to lose the game. Hence, the material function for the king is as follows:

$$M(k) = 2[M(r) + M(h) + M(c) + M(e) + M(a)] + 5M(p) + c = 26.42 + c$$

where c is a small positive constant. The following table summarises the material values of the different pieces.

King	Chariot	Horse	Cannon	Elephant	Advisor	Pawn
26.42 + c	4.147	1.952	2.012	0.9987	1.600	1.000

Table 5: Material values of pieces

9.2 Position

Huang (2008) reported that pieces are of higher value at strategic positions as the effective utility of the pieces increase, especially applicable to pieces whose next move can reach the palace, compromising the safety of the king. This concept is also the basis of our formula, with which a few safe assumptions have been made:

1. None of the pieces will enter the opponent's palace during middle game since entrance of the palace usually constitutes victory of the game which occurs during end game not middle game.
2. The calculation for the number of steps to the king for the last three rows is neglected as pieces have alternative methods of moving in the direction from their side of the board to the opponent's side, albeit taking fewer steps.
3. The component of the function $O(x)$, $O(\chi_0)$ is actually based on the position of the king being at its original position, rather than the palace as the palace is only a discrete indication of the position of the king.

$O(\chi_0)$ is a component of the function $O(x)$ that distinguishes long-range pieces and short-range pieces, such as the chariot and the cannon and the horse and the pawn respectively.

$$O(x) = \text{"}longitude\text{"} \times \text{"}latitude\text{"} + O(\chi_0)$$

whereby "longitude" and "latitude" are numbers indicated on Figure 2. The assignment of the 'longitude' and 'latitude' values are solely based on the absolute distance to the king.

10									
9									
8									
7									
6									
5									
4									
3									
2									
1									
	1	2	3	4	5	4	3	2	1

Table 2: Values of Longitude and Latitude

For horses and pawns:

$$O(\chi_0) = \frac{1}{\dfrac{\min(no.\ of\ \ steps\ to\ king's\ \ position) + \max(no.\ of\ \ steps\ to\ king's\ \ position)}{2}}$$

which is dynamic and varying throughout the game, whereas for rooks and cannons:

$$O(\chi_0) = \frac{M(p)}{M(piece)}$$

$$\therefore O(\chi_0)\ \ for\ rook = \frac{1.000}{2.012} = 0.497$$

$$\therefore O(\chi_0)\ \ for\ cannon = \frac{1.000}{4.147} = 0.241$$

9.3 Flexibility

Apart from material or positional value, other factors, i.e. flexibility must also been considered. Flexibility, although extremely difficult to compute since it is dynamic, is extremely vital in the chess board, especially to Horses and to a smaller extent, Elephants. The flexibility functions of all the other pieces can be calculated as follows:

$$F(x) = \sqrt{\frac{current(\max(capture))}{\max(capture)} \times \frac{current(\max(moves))}{\max(moves)}}$$

9.4 Relationship

There are many instances in the middle game of Chinese Chess when having a certain pair of pieces may be more advantageous as compared to another pair with a higher material value. These phenomena in the relationship function will be accounted.

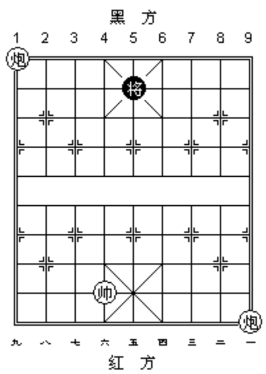
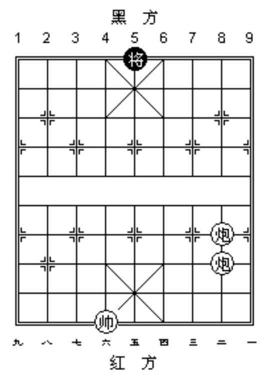
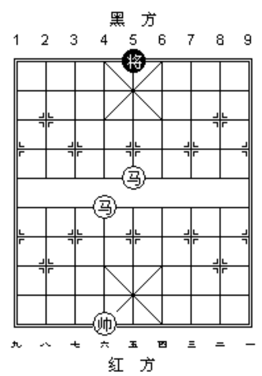
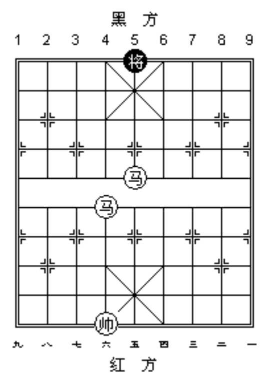
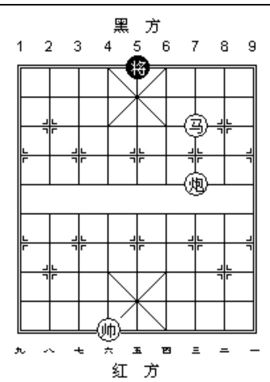
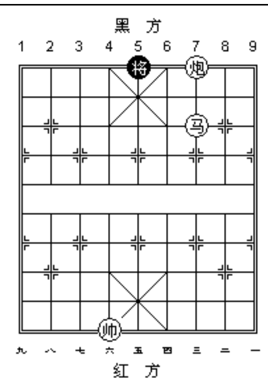
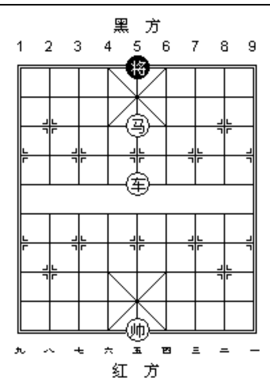
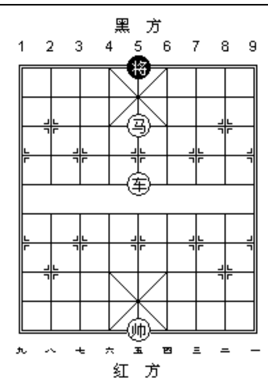
The relationship of the pieces measures the material value of a pair of pieces collectively as a group unit rather than as individual pieces. Therefore, it cannot be taken into account separately from the material; instead it should be calculated together with the material value of the pieces. A detailed description on how the relationship function is calculated with the material function is given in the next sub-section.

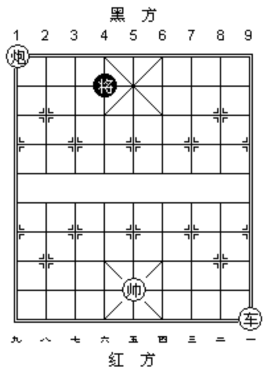
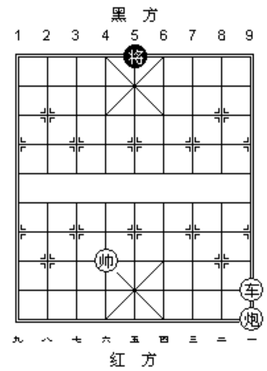
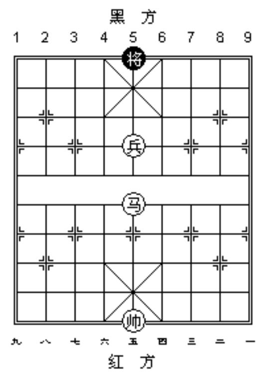
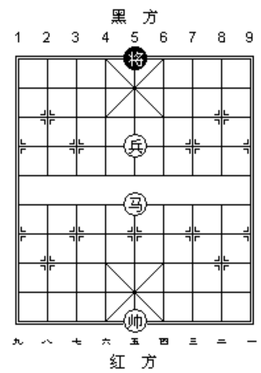
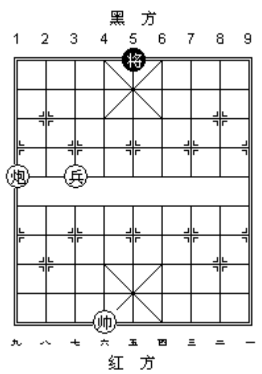
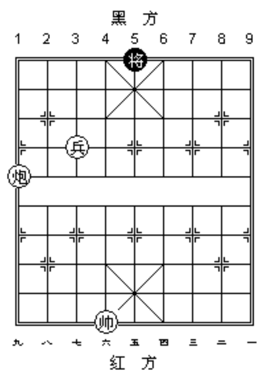
Although the relationship between pieces may not seem important in the middle game, it should be taken into account during moves when the exchange of pieces takes place. In our calculations, all other pieces are assumed to be absent in the sub-section of the chessboard where there is interaction between the pieces Hence, it neglects the probability whereby the pieces are unrestricted. The general formula is found to be:

$$R(x) = \sqrt{\max(moves) \times \max(captures)}$$

As the main purpose of calculating relationship is to find out the effectiveness of the "cooperation" of the pieces, the main indifference lies in the cannon and the horse, for which both attacking pieces have close material value. Respectively, *R(x)*has been calculated:

Pieces	Configuration		*R(x)*
	Maximum moves	Maximum captures	

Double Cannon			$R(x)=\sqrt{\max(moves)\times\max(captures)}$ $=\sqrt{34\times 8}$ $=16.49$
Double Horse			$R(x)=\sqrt{\max(moves)\times\max(captures)}$ $=\sqrt{10\times 10}$ $=10$
Cannon Horse			$R(x)=\sqrt{\max(moves)\times\max(captures)}$ $=\sqrt{14\times 15}$ $=14.49$
Rook Horse			$R(x)=\sqrt{\max(moves)\times\max(captures)}$ $=\sqrt{22\times 22}$ $=22$

Rook Cannon			$R(x)=\sqrt{\max(moves)\times\max(captures)}$ $=\sqrt{17\times 24}$ $=20.20$
Horse Pawn			$R(x)=\sqrt{\max(moves)\times\max(captures)}$ $=\sqrt{11\times 11}$ $=11$
Cannon Pawn			$R(x)=\sqrt{\max(moves)\times\max(captures)}$ $=\sqrt{9\times 20}$ $=13.42$

Hence, it is observed that:

Double Cannon > Horse Cannon > Double Horse

Rook Horse > Rook Cannon

Pawn Cannon > Horse Cannon

9.5 Indicative Function

The above four discussed factors are instrumental and vital in calculating an accurate indicative function. Since *R(x)* and *M(x)* both measures the ability of the piece, individually or collectively, their product can be square-rooted and multiplied by the other variables as the other variables do not measure any common sub-variables:

$$i(x) = \left[\sum \sqrt{M(x) \times R(x)}\right] \times O(x) \times F(x)$$

whereby *M(x)* and *R(x)* measures only for the pieces that are relevant.

The *overall 'material' function* denotes the $\sqrt{M(x) \times R(x)}$ and not the material function $M(x)$.

During the calculation of the indicative function, the overall material function of the pieces is taken, rather than the material function or the relationship function separately. For example, when considering the rook and the horse, the overall 'material' value of the 2 pieces is $\sqrt{M(x) \times R(x)}$ of the 2 pieces. The total overall 'material' value of a player to be used in the indicative function is the sum of all the overall 'material' value of all the pieces.

10. Conclusion

10.1 Conclusion

In conclusion, our objectives have been met and answered our research questions.

The various factors that affect the middle game of the Chinese Chess have been determined and used to create the indicative function $i(x)$, which is mathematically proven to be accurate. Our indicative function is determined by several factors: the material value of the pieces, the relationship between pieces, the flexibility of pieces and the position of the pieces. From these factors, the advantage of a given player at a certain instance during the middle game of Chinese chess can be computed. Also, the computed advantage of a player takes a numerical value and is thus easy to manipulate in order to formulate a series of steps that would allow a player to win the middle game of Chinese chess. Although an algorithm to compute the indicative function directly from the chessboard has yet to be computed, an Excel file which serves the same function has been created in lieu, albeit with human input.

10.2 Extension

Although most of the objectives have been met, several extensions can be added on to our projects based on the limitations encountered during the course of work.

1. Although an Excel file that can compute the indicative function has been creared, it still requires substantial human input. It can be refined further which will minimize the human input that is required to compute the indicative function.
2. Throughout the calculations, several assumptions have been made. Our assumptions can be verified further through deeper research into this area.
3. The factors that make up our indicative function are the major factors that affect the middle game. However, there are also several intricacies of game play that may affect the indicative function slightly. These minor factors can also be considered so as to ensure the accuracy of the indicative function.
4. Other areas can be explored in order to improve the indicative functions such as ways to speed up calculations by ignoring probability trees that are insignificant.

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13. Appendix

1. Rook: Known as 车, it can move in a straight line horizontally and vertically along empty pints similar to International Chess. To capture an opponent's piece, it simply stops at the spot the opponent's piece previously occupied.
2. Horse: Known as 马, it moves one space horizontally or vertically then one space diagonally outwards. When the first space the horse would move to is occupied, its movement in that direction will be obstructed.
3. Elephant: Known as 象 or 相, it moves two spaces diagonally. If the first space diagonal from the elephant is occupied, the elephant's movement in that direction will be obstructed. The elephant cannot exceed the boundaries of the "river".
4. Advisor: Known as 士 or 仕, this piece is restricted to the palace. It moves one point to another only along the diagonal lines in the palace.
5. King: Known as 帅 or 将, this piece is restricted to the palace and it moves horizontally or vertically along the lines in the palace. The king must not be in line with the opponent's king when there are no pieces between the kings.
6. Cannon: Known as 炮, this piece moves exactly like the rook. However, to capture an opponent's piece, it must first "jump" over one obstructing or intervening piece.
7. Pawns: Known as 卒 or 兵, this piece moves only vertically forward before crossing the "river". After it crosses the "river", this piece can move and capture pieces both horizontally and vertically forward. However, this piece can never move backwards.
8. Other than the cannon, all pieces capture in the same mode as the rook.
9. Perpetual checks and perpetual chases are forbidden.